Learn DU Make IT BIG!

All The Best

For Your Exams



[This question paper contains 5 printed pages]		
Your Roll No. (9)	. 2019	
Sl. No. of Q. Paper	: 7462 J	learndu.in
Unique Paper Code	: 32351102 - OC	
Name of the Course	: B.Sc.(Hons.) Mathematics	
Name of the Paper	: Algebra	
Semester	: I	
Time : 3 Hours	Maximum Marks : 75	

Instructions for Candidates :

- Write your Roll No. on the top immediately on receipt of this question paper.
- (ii) Attempt any **two** parts from each questions.
- (iii) All questions are compulsory.
- (a) Find the polar representation for the complex number

 $z = 1 - \cos 4 + i \sin a$, $a \in [0, 2\pi)$

(b) Solve the equation $(2 - 3i)z^6 + 1 + 5i = 0.$

(c) Compute
$$z^{n} + \frac{1}{z^{n}}$$
, if $z + \frac{1}{z} = \sqrt{3}$. 6

P.T.O.



7462

- **2.** (a) Define ~ on \mathbb{Z} by a~b if and only if $2a + 3b \stackrel{6}{=} \frac{16}{5n} \frac{16}{5n}$ for some integer n. Prove that ~ defines an equivalence relation on \mathbb{Z} .
 - (b) Define $f: \mathbb{Z} \to \mathbb{Z}$ by $f(x) = 3x^3 x$.
 - (i) Is fone-to-one?

(ii) Is fonto?

Justify each answer.

(c) Show that the open intervals (0, 1) and (1, 2)

6

6

- have the same cardinality.
- (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that 17369x + 5472y = 1.
 - (b) (i) Show that $3^6 \equiv 1 \pmod{7}$ and hence evaluate $3^{60} \pmod{7}$.
 - (ii) Find all integers x(mod 12) that satisfy $9x \equiv 3(mod 12)$.
 - (c) Use the Principle of Mathematical Induction to prove 2²ⁿ-1 is divisible by 3, ∀n≥1.
- (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_{1} + 3x_{2} + x_{3} = 1$$

- 4x₁ - 9x₂ + 2x₃ = -1
- 3x₂ - 6x₃ = -3
2



(b) Let $A = \begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$. Show that the

equation Ax = b may not be consistent for

every
$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$
. Also describe the set of all

vectors b for which Ax = b is consistent.

6.5

(c) Determine h and k such that the solution set of the given system 6.5

 $x_1 + 3x_2 = k$ $4x_1 + h x_2 = 8$

- (i) is empty.
- (ii) contains a unique solution.
- (iii) contains infinitely many solutions.
- 5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is B₂S₃ + H₂O → H₃BO₃ + H₂S.

Balance the chemical equation using the vector equation approach. 6.5

3

P.T.O.

7462

อ learndu.in

(b) Find the value of h for which the following vectors are linearly dependent. Also find a linear dependence relation among them. 6.5

$$\mathbf{v}_1 = \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4\\5\\6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2\\1\\h \end{bmatrix}$$

(c) A linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ first performs a vertical shear that maps e_1 into $e_2 - 2e_1$, leaves the vector e_2 unchanged and then reflects point through the line $x_2 = x_1$

(i) Find Matrix A such that T(x) = Ax, $x \in \mathbb{R}^2$.

(ii) Find x such that
$$T(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
. 6.5

6. (a) Given :

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

- (i) Show that the matrix A is row equivalent to I_3 .
- (ii) Find inverse of A and hence find inverse of A^T.
 6.5

7462

(b) Find a basis for column space for the matrix A

6.5

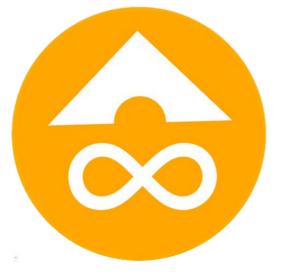
learndu.in

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$$

(c) Is $\lambda = 4$ an eigen value of the matrix A?

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

If so, find eigen space of A corresponding to eigen value $\lambda = 4$. 6.5



Join Us For University Updates









Learn_DU

