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| [This question paper contains 5 printed pages] |                               |            |
|--|-------------------------------|------------|
| Your Roll No. (9)                              | . 2019                        |            |
| Sl. No. of Q. Paper                            | : 7462 J                      | learndu.in |
| Unique Paper Code                              | : 32351102 - OC               |            |
| Name of the Course                             | : B.Sc.(Hons.)<br>Mathematics |            |
| Name of the Paper                              | : Algebra                     |            |
| Semester                                       | : I                           |            |
| Time : 3 Hours                                 | Maximum Marks : 75            |            |

#### **Instructions for Candidates :**

- Write your Roll No. on the top immediately on receipt of this question paper.
- (ii) Attempt any **two** parts from each questions.
- (iii) All questions are compulsory.
- (a) Find the polar representation for the complex number

 $z = 1 - \cos 4 + i \sin a$ ,  $a \in [0, 2\pi)$ 

(b) Solve the equation  $(2 - 3i)z^6 + 1 + 5i = 0.$ 

(c) Compute 
$$z^{n} + \frac{1}{z^{n}}$$
, if  $z + \frac{1}{z} = \sqrt{3}$ . 6

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- **2.** (a) Define ~ on  $\mathbb{Z}$  by a~b if and only if  $2a + 3b \stackrel{6}{=} \frac{16}{5n} \frac{16}{5n}$  for some integer n. Prove that ~ defines an equivalence relation on  $\mathbb{Z}$ .
  - (b) Define  $f: \mathbb{Z} \to \mathbb{Z}$  by  $f(x) = 3x^3 x$ .
    - (i) Is fone-to-one?

(ii) Is fonto?

Justify each answer.

(c) Show that the open intervals (0, 1) and (1, 2)

6

6

- have the same cardinality.
- (a) Define relatively prime integers. Show that 17,369 and 5,472 are relatively prime. Hence, find integers x and y such that 17369x + 5472y = 1.
  - (b) (i) Show that  $3^6 \equiv 1 \pmod{7}$  and hence evaluate  $3^{60} \pmod{7}$ .
    - (ii) Find all integers x(mod 12) that satisfy  $9x \equiv 3(mod 12)$ .
  - (c) Use the Principle of Mathematical Induction to prove 2<sup>2n</sup>-1 is divisible by 3, ∀n≥1.
- (a) Write the solution set of the given system of equations in parametric vector form. 6.5

$$x_{1} + 3x_{2} + x_{3} = 1$$
  
- 4x<sub>1</sub> - 9x<sub>2</sub> + 2x<sub>3</sub> = -1  
- 3x<sub>2</sub> - 6x<sub>3</sub> = -3  
2



(b) Let  $A = \begin{pmatrix} 1 & 3 & 4 \\ -4 & 2 & -6 \\ -3 & -2 & -7 \end{pmatrix}$ . Show that the

equation Ax = b may not be consistent for

every 
$$b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$
. Also describe the set of all

vectors b for which Ax = b is consistent.

6.5

(c) Determine h and k such that the solution set of the given system 6.5

 $x_1 + 3x_2 = k$  $4x_1 + h x_2 = 8$ 

- (i) is empty.
- (ii) contains a unique solution.
- (iii) contains infinitely many solutions.
- 5. (a) Boron sulphide reacts violently with water to form boric acid and hydrogen sulphide gas. The unbalanced equation is B<sub>2</sub>S<sub>3</sub> + H<sub>2</sub>O → H<sub>3</sub>BO<sub>3</sub> + H<sub>2</sub>S.

Balance the chemical equation using the vector equation approach. 6.5

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(b) Find the value of h for which the following vectors are linearly dependent. Also find a linear dependence relation among them. 6.5

$$\mathbf{v}_1 = \begin{bmatrix} 1\\2\\3 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 4\\5\\6 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2\\1\\h \end{bmatrix}$$

(c) A linear transformation  $T: \mathbb{R}^2 \to \mathbb{R}^2$  first performs a vertical shear that maps  $e_1$  into  $e_2 - 2e_1$ , leaves the vector  $e_2$  unchanged and then reflects point through the line  $x_2 = x_1$ 

(i) Find Matrix A such that T(x) = Ax,  $x \in \mathbb{R}^2$ .

(ii) Find x such that 
$$T(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
. 6.5

6. (a) Given :

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$$

- (i) Show that the matrix A is row equivalent to  $I_3$ .
- (ii) Find inverse of A and hence find inverse of A<sup>T</sup>.
  6.5

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(b) Find a basis for column space for the matrix A

6.5

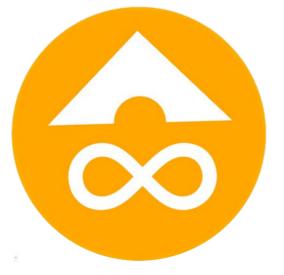
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$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$$

(c) Is  $\lambda = 4$  an eigen value of the matrix A?

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 1 \\ -3 & 4 & 5 \end{bmatrix}$$

If so, find eigen space of A corresponding to eigen value  $\lambda = 4$ . 6.5



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